B.Sc III Paper V Notes 2.

Pointwise Convergence

Suppose $\{f_n\}$, $n = 1, 2, 3$ ---- is a Sequence of functions defined on an interval I, $a \leq x \leq b$. To each point $\xi \in I$, there corresponds a Sequence of numbers $\{f_n(\xi)\}$ with terms

$$f_1(\xi),\ f_2(\xi),\ f_3(\xi), \ldots$$

Let us suppose that the Sequences of numbers $\{f_n(\xi)\}$ converges for every $\xi \in I$

Let $\{f(\xi)\}$ Converge to $f(\xi)$
Let the Sequence at (all) points ξ, η, ζ --- of I converges to
$$f(\xi),\ f(\eta),\ f(\zeta) \ldots \quad (1)$$

Now we define. A real valued

function f, with domain I and range the Set defined by (1) so that its value $f(\eta)$ for $\eta \in I$ is

$\lim \{f_n(\eta)\}$

Thus

$$f(x) = \lim_{n \to \infty} \{f_n(x)\} \quad \forall x \in I$$

The function f, so defined, is referred to as the limit or the pointwise limit of the sequence $\{f_n\}$ on $[a,b]$, and the sequence $\{f_n\}$ is said to be pointwise convergent to f on $[a,b]$

Uniform Convergence on An interval.

A sequence of function $\{f_n\}$ is said to converge uniformly on an interval $[a,b]$ to a function f if for any $\epsilon > 0$ and for all $x \in [a,b]$ there exists an integer N (independent of x but dependent on ϵ). Such that $\forall x \in [a,b]$

$$|f_n(x) - f(x)| < \epsilon \quad \forall n \geq N \quad \text{--- } (1)$$

It is clear that every uniformly convergent sequence is pointwise convergent and uniform limit function is same as the pointwise limit function.